6/2/2020

4. Direct And Bending Stresses

Introduction: –

The member is under two stresses one is due to bending moment and another one is due to direct load.

Eccentric load: –

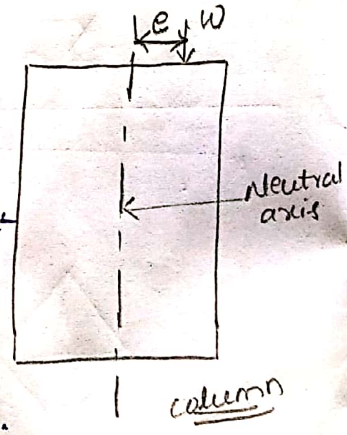
The load acting away from the neutral axis is called "eccentric load".

Eccentricity (e): –

The distance between Neutral axis (or) Centroidal axis and the eccentric load is called "eccentricity".

The eccentric load creates direct stress and bending stress.

Load acting accentrically to one axis: –



Consider a short column subjected to direct load 'w', the line of section which is parallel to axis of the column at a distance of 'e' from the Neutral axis (or) centroidal axis of the column.

Therefore the combined Effect of load 'w' and moment M = load × distance = w×e

Maximum stress $\sigma_{max} = \sigma_d + \sigma_b$

σ_{max} = direct stress + bending stress

Direct stress

$\sigma_d = \frac{W}{A}$ where, W = Eccentric load
A = Area of the section

bending stress

$\sigma_b = \frac{M}{Z} = \frac{M}{(I/y)}$ where, M = moment = $W \times e$
Z = section Modulus = $\frac{I}{y}$

$= \frac{My}{I}$

$\therefore \boxed{\sigma_{max} = \frac{W}{A} + \frac{Wey}{I}}$

$\therefore \boxed{\sigma_{min} = \frac{W}{A} - \frac{Wey}{I}}$

Condition for no tension in the section.

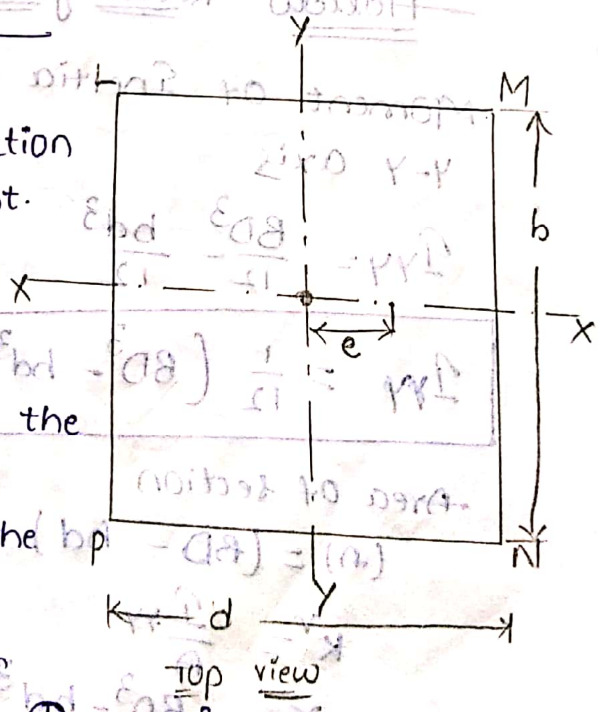
$= e \leq \frac{2K^2}{d}$

Where, 'e' must lies $\frac{2K^2}{d}$ (or) less than $\frac{2K^2}{d}$

d = depth of section

K = radius of gyration $\left[\begin{array}{l} I = AK^2 \\ K = \sqrt{\frac{I}{A}} \end{array}\right]$ $\left[K^2 = \frac{I}{A}\right]$

Rectangular section :—



let a rectangular section LMNP. loaded at a point. distance 'e' along X–X axis.

Consider B = width of the section.

d = depth of the section

Moment of inertia along Y-Y axis is $I_{YY} = \frac{bd^3}{12}$, $I_{XX} = \frac{db^3}{12}$

- Area of section $(A) = b \times d$.

$$K^2 = \frac{I}{A}$$

$$= \frac{\cancel{b}d^{\cancel{3}}}{12 \times \cancel{b} \times \cancel{d}}$$

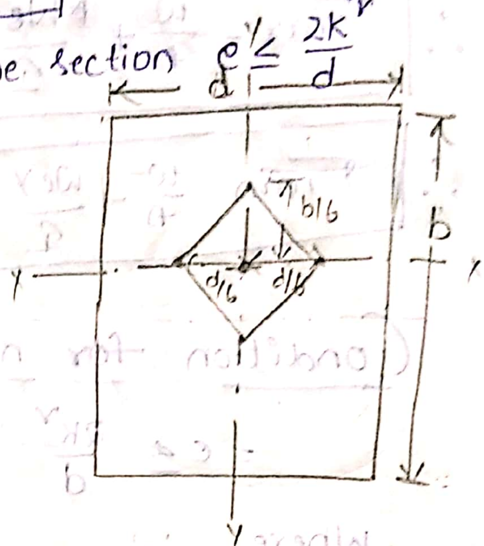
$$\therefore \boxed{K^2 = \frac{d^2}{12}}$$

- for no tension of the section $e \leq \frac{2k^2}{d}$

substituting 'K^2' value

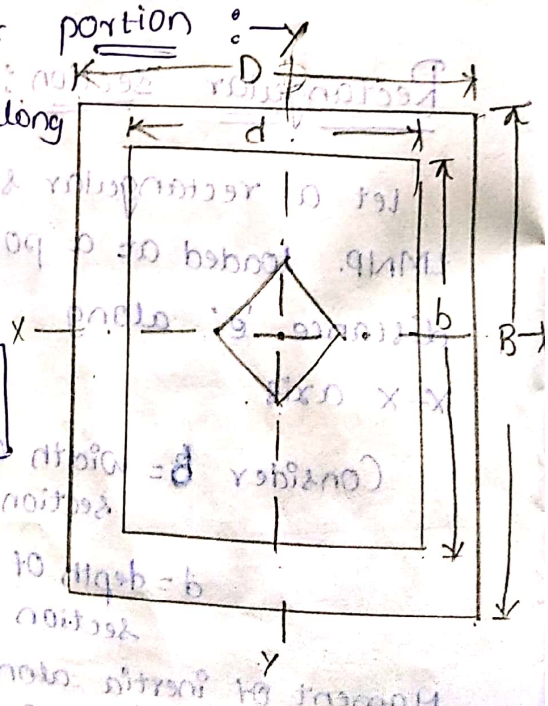
$$e \leq \frac{\cancel{2} \times \frac{d^2}{\cancel{12}}}{\cancel{d}}$$

$$\boxed{e \leq \frac{d}{6}}$$



If the load is placed on any where of inside the rhombus. The reverse space will not occur will not in any part of the entire section.

- **Hollow Rectangular portion:-**



Moment of Inertia along Y-Y axis

$$I_{YY} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$\boxed{I_{YY} = \frac{1}{12}\left(BD^3 - bd^3\right)}$$

- Area of section

$$(A) = (BD - bd)$$

$$K^2 = \frac{I_{YY}}{A}$$

$$K^2 = \frac{BD^3 - bd^3}{12(BD - bd)}$$

For no tension case $e \leq \frac{2K^2}{dD}$

substituting 'K^2' value $e \leq \frac{2K^2}{D}$

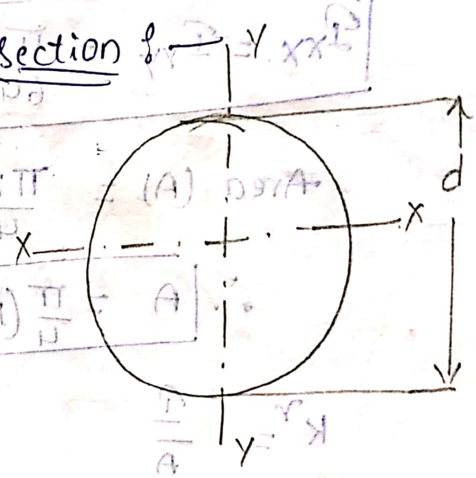
$\therefore$ (In hollow rectangular section the horizontal diagonal of rhombus must be '2e'.)

Hence $2e \leq \frac{2K^2}{D}$

Substitute 'K^2' value $2e \leq \frac{2 \times 2(BD^3 - bd^3)}{12(BD - bd) \cdot D}$

$$\boxed{2e \leq \frac{BD^3 - bd^3}{3(BD - bd)\, D}}$$

For solid circular section :-



Moment of inertia for circular section along x-x and y-y axis

$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$

Area $(A) = \frac{\pi d^2}{4}$

For no tension case $e \leq \frac{2K^2}{d}$

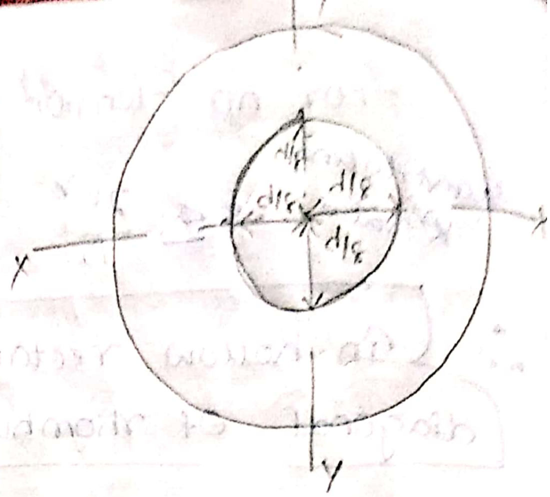
$K^2 = \frac{I}{A}$

$= \frac{\pi d^4}{64} \Big/ \frac{\pi d^2}{4}$

$= \frac{\cancel{\pi} d^{\cancel{4}\,2}}{\cancel{64}_{16}} \times \frac{\cancel{4}}{\cancel{\pi} \cancel{d^2}}$

$\therefore \boxed{K^2 = \frac{d^2}{16}}$

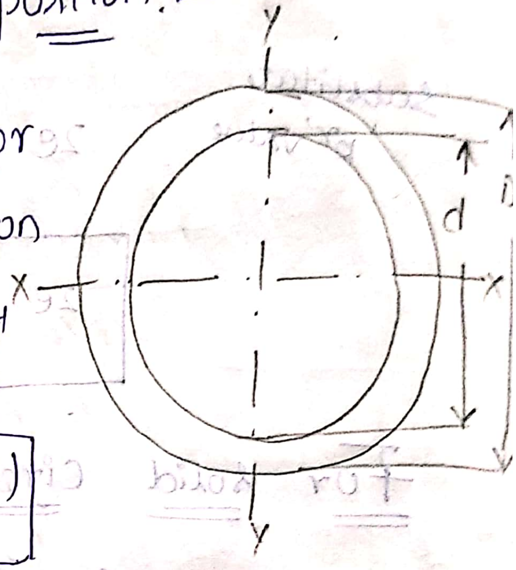
$$e \le \frac{\cancel{2} \times d^{\cancel{2}}}{16 \times \cancel{d}}$$



$$\boxed{e \le \frac{d}{8}}$$

For hollow circular portion:-

Moment of inertia for hollow circular section



$$I_{xx} = I_{yy} = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

$$\boxed{I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)}$$

$$\text{Area } (A) = \frac{\pi D^2}{4} - \frac{\pi d^2}{4}$$

$$\therefore \boxed{A = \frac{\pi}{4}(D^2 - d^2)}$$

$$K^2 = \frac{I}{A}$$

$$= \frac{\frac{\cancel{\pi}}{64}(D^4 - d^4)}{\frac{\cancel{\pi}}{4}(D^2 - d^2)}$$

$$\therefore \boxed{K^2 = \frac{D^4 - d^4}{16(D^2 - d^2)}}$$

$$= \frac{\cancel{(D^2 - d^2)}(D^2 + d^2)}{16\cancel{(D^2 - d^2)}}$$

$$\boxed{(a+b)(a-b) = (a^2 - b^2) \\ (D^2 + d^2)(D^2 - d^2) = (D^4 - d^4)}$$

$$\therefore \boxed{K^2 = \frac{D^2 + d^2}{16}}$$

for no tension case $e \leq \frac{2k^2}{D}$

$$e \leq \frac{2 \times (D^2+d^2)}{16D}$$

$$\therefore \boxed{e \leq \frac{D^2+d^2}{8D}}$$

for hollow circular section $e = 2e$.

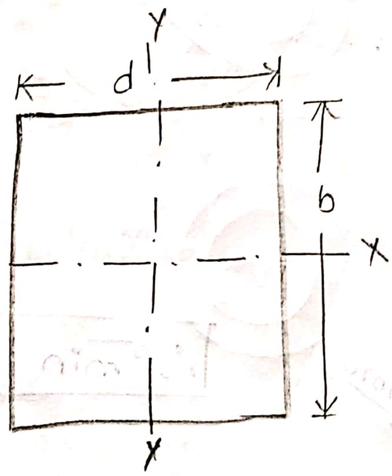
$$2e \leq \frac{2 \times 2k^2}{D}$$

$$2e \leq \frac{2 \times 2 \times (D^2+d^2)}{16D}$$

$$\therefore \boxed{2e \leq \frac{D^2+d^2}{4D}}$$

problems:-

A rectangular strut is 20cm wide and 15cm thick. it carries a load of 60KN. at an eccentricity of 2cm. in a plane bisecting the thickness. Find the maximum and minimum stress intensities at that section.



Sol:- Given data,

width (b) = 20cm = 0.2m

depth (d) = 15cm = 0.15m

load (w) = 60KN

eccentricity (e) = 2cm

$$I_{YY} = \frac{bd^3}{12}$$

$$= \frac{0.2 \times 0.15^3}{12}$$

$$I_{YY} = 5.625 \times 10^{-5}\ m^4$$

$$I_{XX} = \frac{db^3}{12}$$

$$= \frac{0.15 \times 0.2^3}{12} = 1 \times 10^{-4}\ m^4$$

Area of the section (A) = b × d
$= 0.2 \times 0.15$
$= 0.03 m^2$

Centroidal distance from Extreme End (y) = $\frac{d}{2}$

$$y = \frac{0.2}{2}$$

$$y = 0.1 m$$

$$\sigma_{max} = \frac{W}{A} + \frac{Wey}{I}$$

$$= \frac{60}{0.03} + \frac{60 \times 0.02 \times 0.1}{5.625 \times 10^{-5}}$$

$$= 2000 + 2133.33$$

$$= 4133.33 \ KN/m^2$$

$$\therefore \boxed{\sigma_{max} = 4.13 \ N/mm^2}$$

$$\sigma_{min} = \frac{W}{A} - \frac{Wey}{I}$$

$$= \frac{60}{0.03} - \frac{60 \times 0.02 \times 0.1}{5.625 \times 10^{-5}}$$

$$= 2000 - 2133.33$$

$$= -133.33 \ KN/m^2$$

$$\boxed{\therefore \sigma_{min} = -0.133 \ N/mm^2}$$

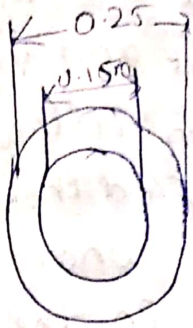
11/2/2020

(2) A short column of hollow cylindrical section 25 cm. Out side diameter 15 cm inside diameter carries a vertical load of 400 KN along one of the diameter planes 10 cm away from the axis of the column. Find the Extreme intensities of stresses and state their nature.

Sol:- Given data,

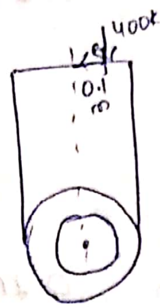
Outside diameter $(D) = 25cm = 0.25m$

Inside diameter $(d) = 15cm = 0.15m$.



Vertical Load $(W) = 400KN = 400\times10^3 N$.

Eccentricity $(e) = 10cm = 0.1m$.



σ_d = direct stress = $\frac{W}{A}$

$$\sigma_d = \frac{400}{\frac{\pi}{4}(D^2-d^2)} = \frac{400}{0.031} = 12732.3 \, KN/m^2$$

$\therefore \sigma_b$ = bending stress = $\frac{Wey}{I}$

where $I = \frac{\pi}{64}(D^4 - d^4)$

$$= \frac{\pi}{64}(0.25^4 - 0.15^4)$$

$$= 1.668\times10^{-4} \, m^4.$$

$$y = \frac{D}{2} = \frac{0.25}{2} = 0.125 \, m$$

$$\sigma_b = \frac{400\times0.1\times0.125}{1.668\times10^{-4}}$$

$$\therefore \sigma_b = 29976.01 \, KN/m^2$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$= 12732.3 + 29976.01$$

$$= 42708.31 \, KN/m^2$$

$$\therefore \boxed{\sigma_{max} = 42.708 \, MN/m^2}$$

$$\sigma_{min} = \sigma_d - \sigma_b$$

$$= 12732.3 - 29976.01$$

$$= -17243.71$$

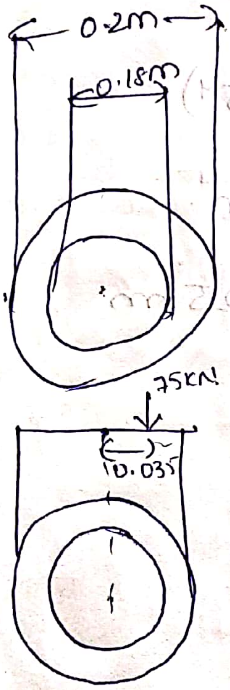
$$\boxed{\sigma_{min} = -17.243 \, MN/m^2}$$

$\therefore \sigma_{max} = 42.708$ MN/m^2 (Tension)

$\therefore \sigma_{min} = -17.243$ MN/m^2 (compression)

(3) A load of 75 KN is carried by a column made up of cast iron the External and internal diameters are 200mm and 180mm if the Eccentricity of the load is 35mm. Find the max and min stress intensities, upto what Eccentricity there is no tensile stress in the column.

Sol:- Given data,



load (W) = 75 KN

External dia (D) = 200mm = 0.2m

Internal dia (d) = 180mm = 0.18m

Eccentricity (e) = 35mm = 0.035m

σ_d = bending stress = $\frac{W}{A}$

$A = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(0.2^2 - 0.18^2)$

$A = 5.96 \times 10^{-3}$ m^2

$\sigma_d = \frac{75}{5.96 \times 10^{-3}} = 12583.8$ KN/m^2

σ_b = bending stress = $\frac{Wey}{I}$

$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(0.2^4 - 0.18^4)$

$\therefore I = 2.700 \times 10^{-5}$ m^4

$y = \frac{D}{2} = \frac{0.2}{2} = 0.1$m

$$\sigma_b = \frac{75 \times 0.035 \times 0.1}{2.700 \times 10^{-5}}$$

$$\sigma_b = 9722.22 \ KN/m^2$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$= 9722.22 + 12583.8$$

$$= 22306.05 \ KN/m^2$$

$$\therefore \boxed{\sigma_{max} = 22.306 \ MN/m^2}$$

$$\sigma_{min} = 12583.8 - 9722.22 = \sigma_d - \sigma_b$$

$$= 2861.58 \ KN/m^2$$

$$\therefore \boxed{\sigma_{min} = 2.86 \ MN/m^2}$$

Upto what Eccentricity there is no tensile stress $\sigma_d = \sigma_b$

$$\frac{W}{A} = \frac{Wey}{I}$$

$$\frac{75}{5.96 \times 10^{-3}} = \frac{75 \times e \times 0.1}{2.700 \times 10^{-5}}$$

$$e = \frac{75 \times 2.700 \times 10^{-5}}{75 \times 0.1 \times 5.96 \times 10^{-3}}$$

$$\therefore \boxed{e = 0.045 \ m} \text{ or } \boxed{e = 45 \ mm}$$

Q) A short column of 20cm External diameter and 15cm internal diameter when subjected to a load the stress measurements indicate that the stress varies from 150 MN/m^2. compressive at one end and 25 MN/m^2 tensile on other End Estimate the load and distance of the line of action from the axis of the column

Sol:- Given data,

External diameter (D) = 20cm = 0.2m

Internal diameter (d) = 15cm = 0.15m

σ_{max} = tensile stress = 25 MN/m^2

σ_{min} = compressive stress = -150 MN/m^2

W = ? , e = ?

$\sigma_{max} = \sigma_d + \sigma_b$

$-150 = \frac{W}{A} + \frac{Wey}{I}$ — ①

$\sigma_{min} = \sigma_d - \sigma_b$

$25 = \frac{W}{A} - \frac{Wey}{I}$ — ②

solving ① & ②

$2\left(\frac{W}{A}\right) = -125.$

$\frac{W}{A} = -62.5$

$\frac{W}{\frac{\pi}{4}(D^2-d^2)} = -62.5$

$W = -62.5 \times 0.013$

$\therefore W = -0.859 \ MN/m^2$

'W' sub in ①

$-150 = \frac{-0.859}{0.013} + \frac{(-0.859) \times e \times 0.1}{5.36 \times 10^{-5}}$

$-150 = -66.07 + \frac{(-0.0859)e}{5.36 \times 10^{-5}}$

$\frac{-0.0859\,e}{5.36 \times 10^{-5}} = -150 + 66.07$

$-0.0859 \times e = -836.93 \times 5.36 \times 10^{-5}$

$\therefore \boxed{e = +0.054 \text{ m}}$

Wind pressure on Chimney:—

Chimney's are tallest structures subjected to horizontal wind pressure due to horizontal wind force.

The base of the chimney subjected to bending moment the corresponding stress is bending stress

The self weight (or) dead load of chimney creates the direct stress.

We know that

direct stress $(\sigma_d) = \frac{W}{A}$

Bending stress $\sigma_b = \frac{M}{Z}$.

The cross-section of the chimney may be hollow, square and circular.

The cross-section may be uniform througho-ut the height (or) tapering towards the height.

The horizontal wind force $\boxed{P = K p A_p}$

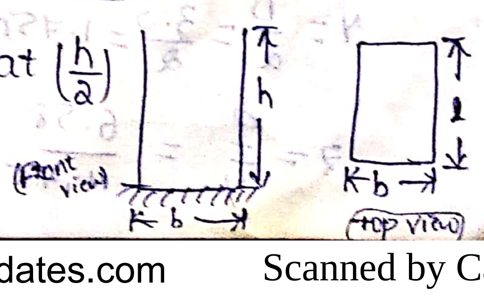
where, P_{wf} = wind force.

K = coefficient of wind resistance

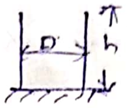
P_w = Intensity of wind pressure

A_p = projected area (or) area of projection.

* Coefficient of wind resistance depends upon shape (a) For square and rectangular chimney's of uniform cross-section $\boxed{K=1}$
* 'P_{wf}' The wind force acts at $\left(\frac{h}{2}\right)$
* projected area $A_p = b \times h$



For circular chimney $K = \frac{2}{3}$

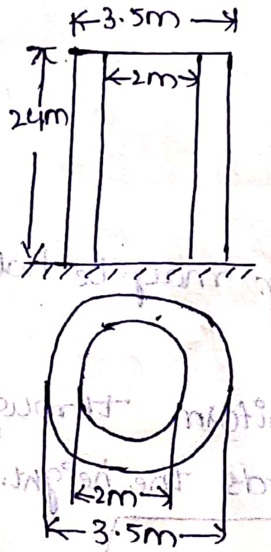


projected area $A_p = D \times h$

Problems

① A masonry chimney 24m height, uniform circular section of 3.5m External diameter and 2m internal diameter. subjected to horizontal wind pressure of $1 KN/m^2$ of projected area. Find the max. and min stress intensities at the base of the section. If the self weight of masonry is $22 KN/m^3$.

Sol:- Given data,



height of chimney (h) = 24m.

External dia (D) = 3.5m

Internal dia (d) = 2m

wind pressure $P_w = 1 KN/m^2$

Self weight of masonry $= 22 KN/m^3$

$$\sigma_{max} = \sigma_d + \sigma_b$$

where, $\sigma_d = \frac{W}{A}$, $\sigma_b = \frac{M}{Z}$

W = Area x height x self weight of masonry

$$= \frac{\pi}{4}(3.5^2 - 2^2) \times 24 \times 22$$

$$W = 3416.16 \text{ KN}$$

$$\sigma_d = \frac{W}{A} = \frac{3416.16}{6.47}$$

$$\therefore \boxed{\sigma_d = 528 \text{ KN}/m^2}$$

$$\sigma_b = \frac{M}{Z} = \frac{M}{(I/y)} = \frac{My}{I} \quad \text{where, } I = \frac{\pi}{64}(D^4 - d^4)$$

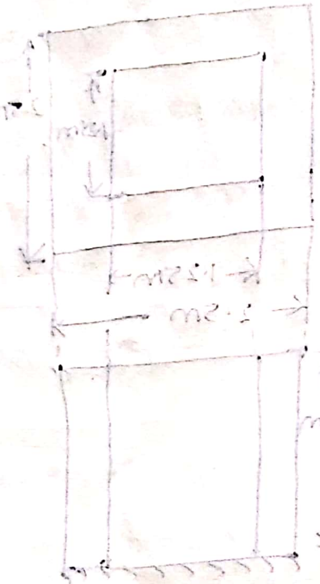
$$y = \frac{D}{2} = \frac{3.5}{2} = 1.75m \qquad = \frac{\pi}{64}(3.5^4 - 2^4)$$

$$Z = \frac{I}{y} = \frac{6.58}{1.75} = 3.76m^3 \qquad \boxed{I = 6.58 m^4}$$

M = Bending moment = force × distance $= P_{wf} \times \frac{h}{2}$

Where $P_{wf} = K P_w \cdot A_p$

for circular section $K = \frac{2}{3}$, $P_w = 1\,KN/m^2$, $A_p = D \times h$
$= 3.5 \times 24$
$= 84\,m^2$



$\therefore P_{wf} = \frac{2}{3} \times 1 \times 84$

$P_{wf} = 56\,KN.$

$M = P_{wf} \times \frac{h}{2}$

$= 56 \times \frac{24}{2}$

$M = 672\,KN.m$

$\sigma_b = \frac{M}{Z} = \frac{672}{3.76}$

$\therefore \boxed{\sigma_b = 178.72\,\frac{KN}{m^2}}$

$\sigma_{max} = \sigma_d + \sigma_b$

$= 528 + 178.72$

$\therefore \boxed{\sigma_{max} = 706.72\,KN/m^2}$

$\sigma_{min} = \sigma_d - \sigma_b$

$= 528 - 178.72$

$\therefore \boxed{\sigma_{min} = 349.28\,KN/m^2}$

A square chimney 24m height. has an opening of 1.25m × 1.25m inside. The External dimensions are 2.5m × 2.5m. The horizontal intensity of wind pressure is 1.3 KN/m². and self weight of masonry is 22 KN/m³. Calculate the max and min stress intensities at the base of the chimney.

Sol:- Given data,

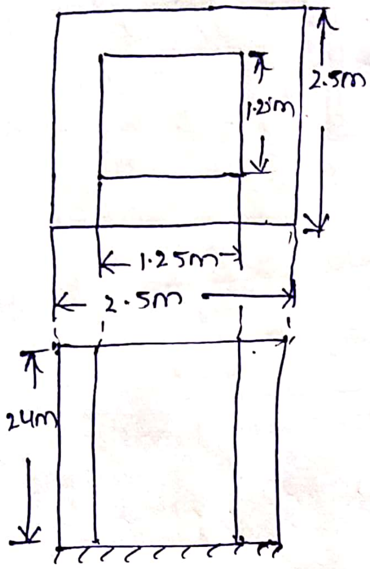
height (h) = 24m

Wind pressure (Pw) = $1.3 kN/m^2$

Inside dimensions = 1.25 x 1.25m

Outside dimensions = 2.5 x 2.5m

self weight of masonry = $22 kN/m^3$



$$Area\,(A) = (A^2 - a^2)$$

$$= (2.5^2 - 1.25^2)$$

$$\therefore A = 4.68\, m^2$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$= \frac{W}{A} + \frac{M}{Z}$$

W = Area x height x selfweight

$$= \boxed{4.68 \times 24 \times 22}$$

$$\therefore W = 2471.04\, kN$$

$$\sigma_d = \frac{W}{A} = \frac{2471.04}{4.68}$$

$$\therefore \boxed{\sigma_d = 528\, kN/m^2}$$

$$\sigma_b = \frac{M}{Z}$$

where $Z = \frac{I}{y}$

$$I = \left(\frac{B^4}{12} - \frac{b^4}{12}\right)$$

$$= \left(\frac{2.5^4}{12} - \frac{1.25^4}{12}\right)$$

$$\therefore \boxed{I = 3.051\, m^4}$$

 www.Jntufastupdates.com

$y = \frac{B}{2} = \frac{2.5}{2}$

$\therefore \boxed{y = 1.25\,m}$

$Z = \frac{I}{y} = \frac{3.051}{1.25}$

$\therefore \boxed{Z = 2.44\,m^3}$

$M = P_{wf} \times \frac{h}{2}$

$= K P_w A_p \times \frac{h}{2}$ (∵ for square section $(K=1)$, $A_p = b \times h = 2.5 \times 24$, $A_p = 60\,m^2$)

$= 1 \times 1.3 \times 60 \times \frac{24}{2}$

$\therefore \boxed{M = 936\ kN\cdot m}$

$\sigma_b = \frac{M}{Z} = \frac{936}{2.44}$

$\therefore \boxed{\sigma_b = 383.60\ kN/m^2}$

$\sigma_{max} = \sigma_d + \sigma_b$

$= 528 + 383.60$

$\therefore \boxed{\sigma_{max} = 911.60\ kN/m^2}$

$\sigma_{min} = \sigma_d - \sigma_b$

$= 528 - 383.60$

$\therefore \boxed{\sigma_{min} = 144.4\ kN/m^2}$

Earth pressure on retaining wall:—

Angle of repose :- It is the natural slope of the material, which they tend to takeup, if not acted upon any external force. The angle of repose is indicated by 'ϕ'.

∴ Lateral earth pressure $\boxed{P = \frac{W_e h^2}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)}$

where, $W_e = \gamma_e$ = unit weight of soil (or) density of soil

h = height of the wall above earth retaining

ϕ = Angle of repose

$\left(\frac{1-\sin\phi}{1+\sin\phi}\right) = k_a$ = coefficient of active earth pressure

The maximum lateral earth pressure at any depth 'y' below the horizontal top is given by

$$\therefore \boxed{P_y = W_e y\left(\frac{1-\sin\phi}{1+\sin\phi}\right)}$$

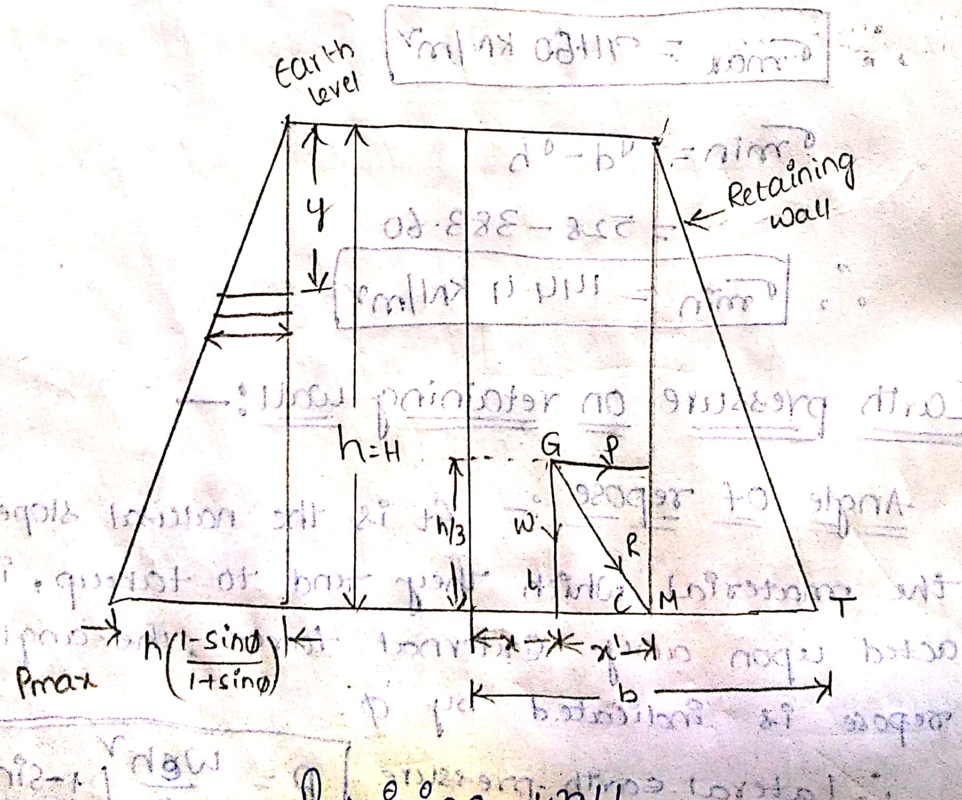
The maximum earth pressure occurs at the bottom of the retaining wall.

$$\boxed{P_{max} \text{ at bottom} = W_e h\left(\frac{1-\sin\phi}{1+\sin\phi}\right)}$$

The centre of gravity 'G' lies at $\left(\frac{h}{3}\right)$ from the bottom of section

$\therefore$ Average earth pressure on retaining wall

$$\therefore \boxed{P_{Avg} = \frac{W_e h}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)}$$



Retaining wall

(8) A masonry retaining wall of trapezoidal section is 10m height and retains earth which is level upto the top. The width at top is 2m and bottom is 8m the exposed face is vertical. Find the max. and min stress intensities at the base. take density of earth (W_e) (or) (γ_e) = 16 kN/m³. density of masonry 24 kN/m³. The angle of repose $\phi = 30°$

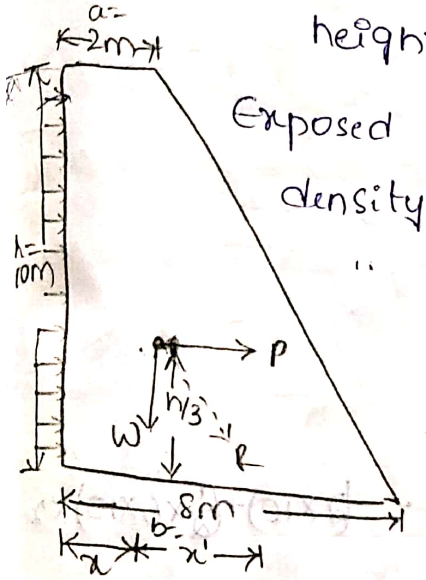
Sol:- Given data,

Top width (a) = 2m

Bottom " (b) = 8m

height (h) = 10m

Exposed face is vertical.



density of earth (W_e) = 16 kN/m³

" " Masonry = 24 kN/m³

Angle of repose (ϕ) = 30°

consider 1m length of retaining wall. weight of masonry (W)

$$W = \left(\frac{a+b}{2}\right) h \times 1 \times \text{unit weight of masonry}$$

$$= \left(\frac{2+8}{2}\right) \times 10 \times 1 \times 24$$

$$\therefore \boxed{W = 1200 \text{ kN}}$$

lateral earth pressure

$$P = \frac{W_e h^2}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

$$= \frac{16 \times 10^2}{2}\left(\frac{1-\sin 30°}{1+\sin 30°}\right)$$

$$\therefore \boxed{P = 266.67 \text{ kN/m}}$$

Let 'G' be the distance from the bottom.

'G' is center of gravity.

$G = \frac{h}{3} = \frac{10}{3} = 3.33m$ from bottom.

Let 'x' be the distance of vertical face to center of gravity.

$$x = \frac{a^2+ab+b^2}{3(a+b)}$$

$$= \frac{2^2+(2\times8)+8^2}{3(2+8)}$$

$$= \frac{84}{30}$$

$$\therefore \boxed{x = 2.8m}$$

(or)

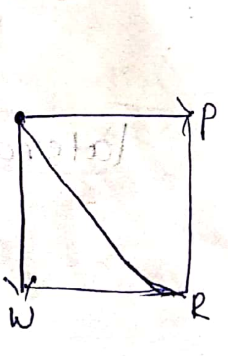
$$(2\times10)\left(\frac{2}{2}\right)+\left(\frac{1}{2}\times6\times10\right)\times\left(2+\frac{1}{3}\times6\right) = (2\times10)+\left(\frac{1}{2}\times6\times10\right)\times x$$

$$(20)+(120) = (20+30)x$$

$$x = \frac{140}{50}$$

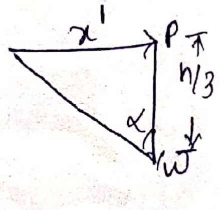
$$\therefore \boxed{x = 2.8m}$$

$$\tan\alpha = \frac{P}{W} \quad -①$$



$$\tan\alpha = \frac{x'}{h/3} \quad -②$$

Equating ① & ②



$$\frac{P}{W} = \frac{x'}{h/3}$$

$$\frac{266.67}{1200} = \frac{x'}{3.33}$$

$$\therefore \boxed{x' = 0.74\,m}$$

$$e = \frac{b}{2} - (x + x') = \left(\frac{8}{2}\right) - (2.8 + 0.74)$$

$$e = 4 - 3.54\,m$$

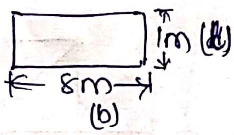
$$\therefore \boxed{e = 0.46\,m}$$

$$M = w \times e$$

$$= 1200 \times 0.46$$

$$\therefore \boxed{M = 552\ KN.m}$$

$$\sigma_d = \frac{W}{A} = \frac{1200\ \text{(weight of masonry)}}{(1 \times 8)\ \text{(Area of base)}}$$ Take $l = 1m$, $b = 8m$



$$= \frac{1200}{8}$$

$$\therefore \boxed{\sigma_d = 150\ KN/m^2}$$

$$\sigma_b = \frac{M}{Z} = \frac{M}{\left(\frac{I}{y}\right)}$$

$$I_{xx} = \frac{bd^3}{12} = \frac{8 \times 1^3}{12} = 0.66\ m^4 \quad , I_{yy} = 42.66\ m^4$$

$$y = \frac{8}{2} = 4m \qquad y = \frac{1}{2} = 0.5m$$

$$\therefore \sigma_b = \frac{552}{\left(\frac{0.66}{4}\right)} \qquad \sigma_b = \frac{552}{\left(\frac{42.66}{0.5}\right)}$$

$$\therefore \boxed{\sigma_b = \cancel{209.09}\ 3345.45\ KN/m^2} \qquad \therefore \boxed{\sigma_b = 6.46\ KN/m^2}$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

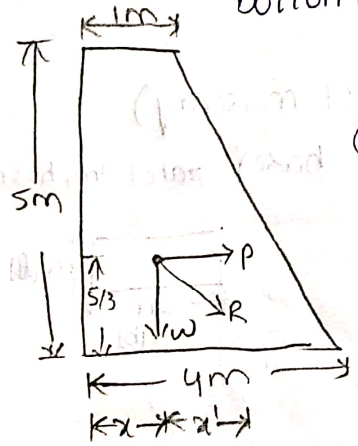
$$= 150 + 6.46 = 156.46\ KN/m^2$$

$$\sigma_{min} = \sigma_d - \sigma_b$$

$$= 150 - 6.46 = 143.54\ KN/m^2$$

② A masonry retaining wall is 1m wide at top and 4m at bottom. and retains water level at its top. the wall is 5m height. test stability of wall against (i) Tension (ii) Crushing (iii) sliding (iv) Overturning. The weight of ~~masonry~~ concrete is $24 kN/m^3$. bearing capacity of soil is 240 kN/m^3. coefficient of friction $(\mu) = 0.6$.

Sol:- Given data,



Top width $(a) = 1m$

bottom width $(b) = 4m$

height $(h) = 5m$.

Coefficient of friction $(\mu) = 0.6$

unit weight of ~~masonry~~ concrete $= 24 kN/m^3$

bearing capacity of soil $= 240 kN/m$

consider 1m length of retaining wall

Weight of masonry $(W) = \left(\frac{a+b}{2}\right) h \times 1 \times$ unit weight of concrete

$$= \left(\frac{1+4}{2}\right) \times 5 \times 1 \times 24$$

$$\therefore \boxed{W = 300 kN}$$

Hydrostatic pressure $(P) = \frac{W_{water} h^2}{2}$

$$= \frac{9.81 \times 5^2}{2}$$

$$\therefore P = 125 kN/m$$

Center of gravity $(G) = \frac{h}{3}$

$$= \frac{5}{3}$$

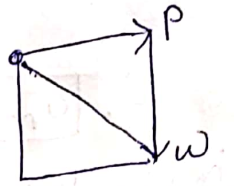
$$\therefore \boxed{G = 1.66m}$$

Let 'x' distance from Vertical face to center of gravity

$$x = \frac{a^2+ab+b^2}{3(a+b)}$$

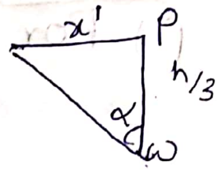
$$= \frac{1^2+(1\times4)+4^2}{3(1+4)}$$

$\therefore \boxed{x = 1.4m}$



$\tan\alpha = \frac{P}{W}$ — ①

$\tan\alpha = \frac{x'}{h/3}$ — ②.



Equating ① = ②

$$\frac{P}{W} = \frac{x'}{h/3}$$

$$\frac{125}{300} = \frac{x'}{1.66}$$

$$x' = \frac{125\times1.66}{300}$$

$\therefore \boxed{x' = 0.69\ m}$

$$e = -\frac{b}{2} + (x+x')$$

$$= -\frac{4}{2} + (1.4+0.69)$$

$$= -2 + 2.09$$

$\therefore \boxed{e = +0.09\ m} = 90mm$

$M = W\times e$

$= 300\times0.09$

$\boxed{M = 27\ KN.m}$

$\sigma_{max} = \frac{W}{b}\left(1+\frac{6e}{b}\right)$

$= \frac{300}{4}\left(1+\frac{6\times0.09}{4}\right)$

$\therefore \boxed{\sigma_{min} = 85.125\ KN/m^2}$

$\sigma_{min} = \frac{W}{b}\left(1-\frac{6e}{b}\right)$

$= \frac{300}{4}\left(1-\frac{6\times0.09}{4}\right)$

$\therefore \boxed{\sigma_{min} = 64.875\ KN/m^2}$

Check for stability conditions :—

(i) For no tension :—

$e \le \frac{b}{6}$

$0.09 \le \frac{4}{6}$

$0.09 \le 0.66$

Hence it is safe in tension

(ii) For no crushing :

σ_{max} less than bearing capacity of soil

$85.125 < 240$

Hence it is safe in crushing

(iii) For no sliding :—

$= \frac{\mu \times W \text{ (weight of masonry)}}{P}$

$= \frac{0.6\times300}{125}$

$= 1.44$

sliding is must be >1

1.44 > 1

Hence it is safe from sliding.

(iv) Overturning :-

Factor of saftey against overturning

$= \dfrac{\text{restoring moment}}{\text{overturning moment}}$

restoring moment $= W \times (b - x)$

$= 300 \times (4 - 1.4)$

$= 780$ KN·m

Overturning moment $= P \times \left(\frac{h}{3}\right)$

$= 125 \times \frac{5}{3}$

$= 208.33$ KN·m

F.O.S overturning $= \dfrac{780}{208.33}$

$= 3.744$

Overturning is must be > 1

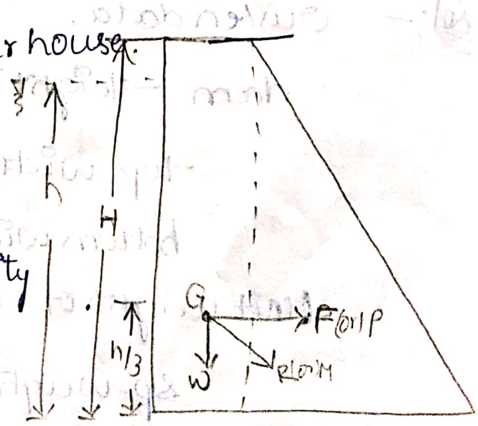
3.744 > 1

Hence, it is safe from overturning.

25/2/2020

Masonry dams:-

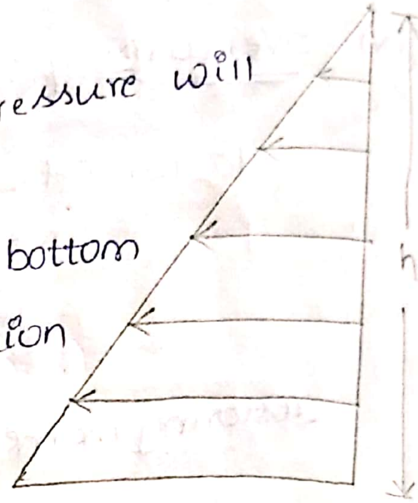
A dam is a structure to store water and to provide head to a power house.



Let 'W' be the weight of dam. acting downwards. through the center of gravity

Let 'w' be the sp.weight of water = 9.81

Water pressure diagram (or) pressure distribution diagram:-



When depth increase pressure will be increase.

The water pressure at bottom of the dam and it's variation is linear and 'zero' at the top.

∴ Total water force acting on dam

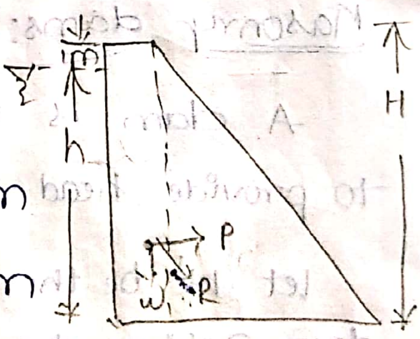
F = Area x Mean pressure.

$$\therefore \boxed{F = \frac{wh^2}{2}}$$

The center of force acts at $\left(\frac{h}{3}\right)$ from the bottom of the dam.

Let 'b' be the base width of the dam.

① A masonary dam is 10m height, it's width at top and bottom is 2m and 6m respectively it's stress values at base the weight of Masonry is 22 kN/m³ and sp. weight of water is 9.81 kN/m³.



Sol:- Given data, free board = 1m

dam Height (H) = 10m

top width (a) = 2m

bottom width (b) = 6m

Unit weight of ~~concrete~~ Masonry = 22 kN/m³

sp. weight of water (w) = 9.81 kN/m³.

Water level height (h) = H-1
=10-1
h = 9m

Consider 1m length of dam. weight of Masonry $W = \left(\frac{a+b}{2}\right) h \times 1 \times$ unit weight of masonry

$= \left(\frac{2+6}{2}\right) \times 10 \times 1 \times 22$

$\therefore \boxed{W = 880 \text{ KN}}$

Hydrostatic pressure $F = \frac{wh^2}{2}$

$= \frac{9.81 \times 9^2}{2}$

$= 397.30 \text{ KN/m}$

Center of gravity $G = \frac{H}{3}$

$= \frac{10}{3}$

$= 3.33m$

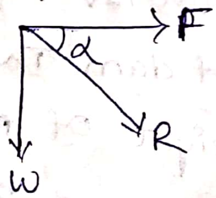
$x = \frac{a^2+ab+b^2}{3(a+b)}$

$= \frac{2^2+(2\times6)+6^2}{3(2+6)}$

$\therefore \boxed{x = 2.166m}$

$\tan\alpha = \frac{F}{w}$ — ①

$\tan\alpha = \frac{x'}{\left(\frac{h}{3}\right)}$ — ②



Equating ① & ②

$\frac{F}{w} = \frac{x'}{h/3}$

$$\frac{397.30}{880} = \frac{x'}{3.33}$$

$$\therefore \boxed{x' = 1.503m}$$

$$e = -\frac{b}{2} + (x + x')$$

$$= -\frac{6}{2} + (2.166 + 1.503)$$

$$\therefore \boxed{e = 0.669}$$

$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{880}{6}\left(1 + \frac{6 \times 0.669}{6}\right)$$

$$\therefore \boxed{\sigma_{max} = 244.78 \ KN/m^2}$$

$$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{880}{6}\left(1 - \frac{6 \times 0.669}{6}\right)$$

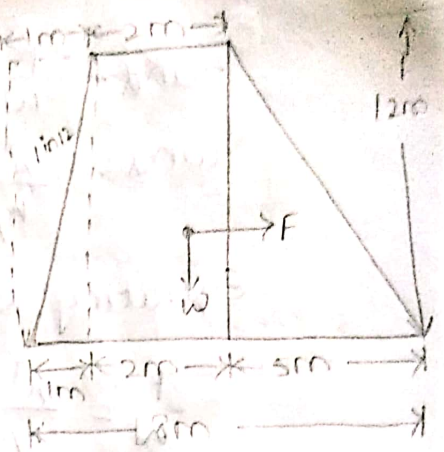
$$\therefore \boxed{\sigma_{min} = 48.54 \ KN/m^2}$$

(2) A masonry dam trapezoidal in section is 12m height it is 2m wide at top and 8m wide at bottom. The face exposed to water has a slope of 1 H to 12V. The water level in dam is upto top of dam. The weight of masonry is 25 KN/m³ sp. weight of ~~masonry~~ water is 9.81 KN/m³. determine the max. and min stress values at base and also check the stability of dam. If coefficient of friction of dam and soil is 0.6.

Sol:- Given data,

top width (a) = 2m

bottom width (b) = 8m



Consider 1m length of dam.

$W_1 = \left(\frac{a+b}{2}\right) h \times 1 \times$ unit weight of masonry

$= \left(\frac{2+8}{2}\right) \times 12 \times 1 \times 25$

$\therefore \boxed{W_1 = 1500 \text{ KN}}$

~~Eq~~ weight of water contained slope section (1 H to 12V)

$W_2 = \frac{1}{2} \times b h \times$ unit weight of water

$= \frac{1}{2} \times 1 \times 12 \times 9.81$

$W_2 = 58.86$ KN

Total weight (W) $= W_1 + W_2$

$= 1500 + 58.86$

$\therefore \boxed{W = 1558.86 \text{ KN}}$

Hydrostatic pressure (F) $= \frac{wh^2}{2}$

$= \frac{9.81 \times 12^2}{2}$

$\therefore \boxed{F = 706.32 \text{ KN/m}}$

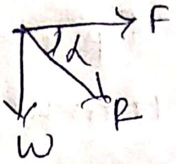
Center of gravity $G = \frac{h}{3}$

$= \frac{12}{3}$

$\therefore \boxed{G = 4m}$

$x = \frac{a^2 + ab + b^2}{3(a+b)} = \frac{2^2 + (2 \times 8) + 8^2}{3(2+8)} = 2.8$ m

$\tan\alpha = \frac{F}{W}$ — ①



$\tan\alpha = \frac{x'}{h/3}$ — ②

Equating ① & ②

$$\frac{F}{W} = \frac{x'}{h/3}$$

$$\frac{706.32}{1558.86} = \frac{x'}{4}$$

$$\therefore \boxed{x' = 1.812\ m}$$

$$e = -\frac{b}{2} + (x + x')$$

$$= -\frac{8}{2} + (2.8 + 1.812)$$

$$\therefore \boxed{e = 0.612}$$

$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{1558.86}{8}\left(1 + \frac{6 \times 0.612}{8}\right)$$

$$\therefore \boxed{\sigma_{max} = 284.29\ kN/m^2}$$

$$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{1558.86}{8}\left(1 - \frac{6 \times 0.612}{8}\right)$$

$$\therefore \boxed{\sigma_{min} = 105.41\ kN/m^2}$$

Check for stability :—

① For no tension

$$e \le \frac{b}{6}$$

$0.612 \le \frac{8}{6}(1.333)$, $0.612 \le 1.33$

Hence there is no tension

For no sliding

$= \frac{\mu \times W}{P}$

$\frac{0.6 \times 1558.86}{706.32}$

$1.32 > 1$

Hence there is no sliding

For overturning

restoring moment $= W(b - x)$

$= 1558.86 \times (8 - 2.8)$

$= 8106.02$ KN.m

overturning moment $= P \times \frac{h}{3}$

$= 706.32 \times \frac{12}{3}$

$= 2825.28$ KN.m

F.O.S overturning $= \frac{\text{restoring moment}}{\text{overturning moment}}$

$= \frac{8106.02}{2825.28}$

$= 2.869 > 1$

Hence it is safe from overturning